SEM-III

USC03STA21

UNIT- III

INDEX NUMBERS

- **Meaning and definition of Index numbers:**

In day to day life changes are observed in the price of a commodity, imports, industrial production, unemployment, etc. as time changes. It is obvious that such changes can neither be uniform nor in the same direction. Variables like population, price of a commodity, industrial production etc. increase with time, while variables like death rate, value of money decrease with time. The analytical study of such changes is essential for future planning. The average being an absolute measure cannot be used to compare the changes like these, and hence some relative measure should be used for a proper comparison of such changes. Index number is such a measure with the help of which relative changes over a period of time can be studied. Index numbers are the indicators which reflect changes over a specified period of time in the values of a variable or a group of variables. According to Spigel, **an index number is a statistical measure designed to show changes in a variable or a group of related variables with respect to time, geographical location or other characteristics.** It may be described as a specialized average designed to measure the change in level of phenomenon with respect to time or place. It is a ratio which shows the changes in the magnitude of a variable over a period of time. For example, if the price of rice in 2009 is Rs.2500 per quintal and it is Rs. 3475 per quintal in 2010 then the ratio of changes in price i.e. index number of the price in 2010 compared to that in 2009 can be given by

Index number = $\frac{Price\ in\ 2010}{price\ in\ 2009} \times 100$

$= \frac{3475}{2500} \times 100 = 138$

Hence it can be concluded that the percentage increase in the price of rice is 38 percent compared to that in 2009. Similarly, index number of industrial production, index number of wages etc. can be constructed.

An index number is a numerical expression showing relative percentage changes in the value of a variable from one period to another. The period for which the index number is calculated is called the current period and the period with which the comparison is made is called the base period.

Index number is also called barometer of the economy of the country.

- **Characteristics of Index number:**

The following are the main characteristics of index numbers:

(1) Index numbers showing relative percentage change in a variable or a group of variables.
(2) Index numbers is a specialized average and hence it shows an average change.
(3) It is a weighted average.
(4) If the variables are expressed in different units, the comparison can be made by index numbers.

- **Uses of Index numbers:**

The main uses of different index numbers can be given as follows:

(1) **Wholesale Price index numbers** indicate the changes in the wholesale prices of commodities. From the study of wholesale price index numbers industrialists, investors etc. get useful guidance to decide their policies. These index numbers are also measure the purchasing power of money. By using this index number the decision regarding the rate of interest, loan reserves credit policies etc. can be taken.

(2) **Index number for industrial production** reveal the progress of industrial sector and give useful guidance for framing industrial policies and tax structure.

(3) **The cost of living index numbers** are useful in deciding dearness allowance (DA), bonus etc. for working class people to protect them against inflation.

(4) **Index numbers of import – export** indicate the balance of foreign trade and the situation of foreign reserves.

(5) **Index numbers of employment** are helpful for deciding the policies to solve the problem of unemployment.

(6) **Index numbers of investment** indicate the trend of the share and the stock market and help us to study the future effects.

(7) **Index numbers of agricultural production** show the progress in the agriculture fields and help the government in framing suitable policies.

(8) **Index numbers of raw materials** provide useful guidance to economists, industrialists, etc. in their policies.

- **Limitations of Index numbers:**

Index numbers are very useful to study the economic changes; however the following limitations of index numbers should be pointed out.

(1) Index numbers measures the changes in certain, phenomenon with respect to some period. This period is known as the base period or base year. The base year should be normal year free from abnormal events like war, famine, flood etc. the factors affecting the economy.

(2) Index number obtained for a particular purpose cannot be applied for other purpose. The cost of living index number for the working class people cannot be applied to higher middle class or rich people.

(3) The selection of commodities, their prices and proper weighing system play vital role in the construction of index numbers. If all these are not selected properly, index numbers do not give the correct picture of situation.

- **Notations and Terminology**

Base Year: The year (period) with which the comparison is made is called the base period. OR the year selected for comparison i.e. the year w.r.to which comparisons are made is called base year. It is denoted by suffix '0'

Current year: The year (period) for which the index number is calculated is called the current period. OR the year for which comparisons are sought or required. It is denoted by suffix '1'.

p_0: Price of a commodity in the base year

p_1: Price of a commodity in the current year

q_0: Quantity of a commodity consumed or purchased during base year

q_1: Quantity of a commodity consumed or purchased in the current year

w: Weight assigned to a commodity according to its relative importance in the group.

I: Simple index number or price relative obtained on expressing current year price as a percentage of the base year price and is given by

I = Price relative = $\frac{p_1}{p_0} \times 100$

P_{01} = Price index number for the current year w.r.to the base year

P_{10} = Price index number for the base year w.r.to the current year

Q_{01} = Quantity index number for the current year w.r.to the base year

Q_{10} = Quantity index number for the base year w.r.to the current year

V_{01} = Value index number for the current year w.r.to the base year

Index number of the base year is 100.

- **Methods of constructing Index numbers:**

(1) Simple (Un-weighted) Aggregate method: This is the simplest of all the methods of constructing index numbers and consists in expressing the total price, i.e. aggregate of prices (of all the selected commodities) in the current year as a percentage of the aggregate of prices in the base year. Thus the price index for current year w.r.to the base year is given by

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

Where $\sum p_1$ is the aggregate of prices (of all the selected commodities) in the current year and $\sum p_0$ is the aggregate of prices in the base year.

Example: from the following data calculate index numbers using Simple Aggregate method.

Commodity	Price (in Rs.)	
	2005	2010
A	162	185
B	256	314
C	257	189
D	132	180

Solution:

Computation of Price Index using Simple Aggregate Method

Commodity	Price (in Rs.)	
	2005 (p_0)	2010 (p_1)
A	162	185
B	256	314
C	257	189
D	132	180
Total	$\sum p_0$ = 807	$\sum p_1$ = 868

$$\therefore P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

$$= \frac{868}{807} \times 100 = 107.56$$

(2) Weighted Aggregate Method:

(a) **Laspeyre's (Price index) Method:**

In this method the base year quantities are taken as weights. The formula for constructing Laspeyre's index number is

$$L = P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

(b) **Paasche's (Price index) Method:**

In this method the current year quantities are taken as weights. The formula for constructing Paasche's index number is

$$P = P_{01}^{\ P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

(c) **Fisher's ideal Index number:**

Fisher's index numbers is the **geometric mean** of the Laspeyre's and Paasche's index numbers. According to this method,

$$P_{01}^{\ F} = \sqrt{P_{01}^{\ L} \times P_{01}^{\ P}} = \sqrt{L \times P} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

- **Tests of consistency of Index numbers:**

Various formulae are used for calculating index numbers. From the statistical point of view the formula for the calculation of index numbers should be such that it should satisfy certain mathematical tests. Prof. Irving Fisher has suggested the following two tests to be satisfied by a good index number

(1) Time Reversal Test

(2) Factor Reversal Test

(1) Time Reversal Test: According to Fisher "the formula for calculating an index number should be such that it gives the same ratio between the one points of comparison the other, no matter which of the two taken as the base". In other words if the base period are interchanged the two index numbers should be reciprocal to each other.

Thus if P_{01} represents the given index number and P_{10}, the number after changing base year and current year, then the Time Reversal Tests demands that $P_{01} \times P_{10} = 1$

Now let us apply Time Reversal Test to different index numbers:

(i) Laspeyre's Index Number:

According to Laspeyre's formula,

$$P_{01}^{\ L} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$$

Interchanging time 0 and 1, we get

$$P_{10}^{\ L} = \frac{\sum p_0 q_1}{\sum p_1 q_1}$$

Now $P_{01}^{\ L} \times P_{10}^{\ L} = \frac{\sum p_1 q_o}{\sum p_0 q_0} \times \frac{\sum p_0 q_1}{\sum p_1 q_1} \neq 1$

So Laspeyre's Index number does not satisfy Time Reversal Test.

(ii) Paasche's Index Number:

According to Paasche's formula,

$$P_{01}^{\ P} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$$

Interchanging time 0 and 1, we get

$$P_{10}{}^{P} = \frac{\sum p_0 q_0}{\sum p_1 q_0}$$

Now $P_{01}{}^{P} \times P_{10}{}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0} \neq 1$

So Paasche's Index number does not satisfy Time Reversal Test

(iii) Fisher's Index Number:

According to Fisher's formula:

$$P_{01}{}^{F} = \sqrt{P_{01}{}^{L} \times P_{01}{}^{P}} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

Interchanging time 0 and 1, we get

$$P_{10}{}^{F} = \sqrt{P_{10}{}^{L} \times P_{10}{}^{P}} = \sqrt{\frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

$$\therefore P_{01}{}^{F} \times P_{10}{}^{F} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}} = \sqrt{1} = 1$$

So, Fisher's Index number satisfies Time Reversal Test.

(2) Factor Reversal Test:

The other test suggested by Fisher is Factor Reversal Test. According to him, "just as each formula should permit the interchange of the two times without giving inconsistent results, so it out to permit interchanging the prices and quantities without giving inconsistent result. i.e. the two results multiplied together should give the value ratio". This implies that if the price and quantity indices are obtained for the same data, same base and current periods and using the same formula, then their product (without the factor 100) should give the true value ratio. Symbolically, we should have (without the factor 100),

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = V_{01}$$

Now let us apply Time Reversal Test to different index numbers:

(i) Laspeyre's Index Number:

According to Laspeyre's formula,

$$P_{01}{}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$$

Interchanging the two factors price (p) and quantity (q), we get

$$Q_{01}{}^{L} = \frac{\sum q_1 p_0}{\sum q_0 p_0} = \frac{\sum p_0 q_1}{\sum p_0 q_0}$$

Now, $p_{01}{}^{L} \times Q_{01}{}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_0 q_1}{\sum p_0 q_0} \neq \frac{\sum p_1 q_1}{\sum p_0 q_0}$

So, Laspeyre's index number does not satisfy Factor Reversal Test.

(ii) Paasche's Index Number:

According to Paasche's formula,

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$$

Interchanging the two factors price (p) and quantity (q), we get

$$Q_{01}^{P} = \frac{\sum q_1 p_1}{\sum q_0 p_1} = \frac{\sum p_1 q_1}{\sum p_1 q_0}$$

Now, $p_{01}^{P} \times Q_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_1 q_1}{\sum p_1 q_0} \neq \frac{\sum p_1 q_1}{\sum p_0 q_0}$

So, Paasche's Index number does not satisfy Factor reversal test.

(iii) Fisher's Index Number:

According to Fisher's formula:

$$P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{P}} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

Interchanging the two factors price (p) and quantity (q), we get

$$Q_{01}^{F} = \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} = \sqrt{\frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}}$$

Now, $p_{01}^{F} \times Q_{01}^{F} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$

So, fisher's index number satisfies Factor reversal test.

Since fisher's index number satisfies both time reversal and Factor reversal tests hence fisher's index number is said to be ideal index number.